Wall Shear Rates in Circular Couette Flow of a Herschel-Bulkley Fluid

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Abstract:
The objective of this work is to study quantitatively the errors introduced by the standard Newtonian and power-law assumptions used in the determination of the material properties of viscoplastic fluids from circular Couette experiments. The steady-state circular Couette flow of a Herschel-Bulkley fluid is solved assuming that the inner cylinder is rotating at constant speed while the outer one is fixed. Analytical solutions are presented for certain values of the power-law exponent. It is shown that the error in the computed wall shear rate, which is insignificant when the diameter ratio is closed to unity, may grow large depending on the diameter ratio and the material parameters.

Zusammenfassung:
Ziel dieser Arbeit ist eine quantitative Untersuchung der Fehler, die in die Ermittlung der Materialeigenschaften viscoplastischer Flüssigkeiten in Rotations-Couette Experimenten durch die üblichen Newtonschen und Potenzgesetzannahmen eingeführt werden. Der stationäre Rotations-Couette-Fluss einer Herschel-Bulkley Flüssigkeit wird gelöst unter der Annahme, dass der innere Zylinder sich mit konstanter Geschwindigkeit dreht, während der Äussere ruht. Für bestimmte Werte des Exponenten im Potenzgesetz werden analytische Lösungen angegeben. Es wird gezeigt, dass der bei Durchmesserverhältnissen nahe Eins bedeutungslose Fehler in der berechneten Wandscherrate in Abhängigkeit vom Durchmesserverhältnis und den Materialkonstanten große Werte annehmen kann.

Résumé:
L'objectif de ce travail est d'étudier quantitativement les erreurs présentées par les suppositions ordinaires newtoniennes et de loi de puissance faites pour la détermination des propriétés matérielles des fluides viscoplastiques des expériences circulaires de Couette. L'écoulement permanent de Couette circulaire d'un fluide de Herschel-Bulkley est résolu supposant que le cylindre intérieur tourne à la vitesse constante tandis que le cylindre extérieur est fixe. Des solutions analytiques sont présentées pour certaines valeurs de l'indice de loi de puissance. On montre que l'erreur du taux de cisaillement calculé au paroi, qui est insignifiant quand le rapport de diamètre est proche de l'unité, peut se développer grande selon le rapport de diamètre et les paramètres matériels.

Key words: circular Couette flow, Herschel-Bulkley fluid, Bingham plastic, viscometry

1 INTRODUCTION

Circular Couette flow is a standard, widely used viscometric flow for many fluids including materials with yield stress, such as paints, cosmetics, drilling fluids and aqueous bentonite suspensions [1], liquid foods [2, 3], granular suspensions [4], cement paste and fresh concrete [5], and semisolid metal slurries [6]. An interesting article discussing the origins of Couette rheometry has been recently published by Dontula et al. [7]. Macosko [8] discusses the underlying assumptions as well as the possible sources of error, such as end effects, wall slip, eccentricities, and viscous heating.

Converting torque measurements in a Couette rheometer to flow curve data, i.e. to the shear stress versus shear rate plot, is an ill-posed inverse problem, known as Couette inverse problem, which becomes more complicated in the case of fluids with yield stress, due to the possible existence of an unsheared region at the fixed cylinder [9]. In the past three decades, the issue of the correct determination of the shear rate has been addressed by various investigators, who proposed some procedures to overcome certain limitations. A nice review including early works on the subject is provided by Estellé et al. [3]. It should be noted that for the characterization of certain industrial suspensions, Couette viscometers with large annular gap are often employed, and, as a result, the narrow gap approximation is not valid. As pointed out by Nguyen and Boger [9], the shear stress and the shear rate are not uniformly distributed within the gap, and more detailed data analysis is required. Industrial suspensions commonly contain a proportion of large particles requiring larger gap sizes to eliminate particle size effects [10]. According to Boger [11], the gap needs to be ten times larger than the largest particle to conduct successful measurements and avoid particle wall friction. In order to extract material parameters from the experimental data, one also needs to pre-specify the rheological constitutive equation. The most commonly used constitutive equations for materials with yield stress are the Bingham plastic, the Herschel-Bulkley and the Casson models [12, 13].

Yeow and co-workers [2, 14] used the Tikhonov regularization method to convert data from wide- as well as narrow-gap Couette viscometers into shear flow curve data for various liquid foods. They concluded that Tikhonov regularization is a practical and reliable method of processing Couette viscometry data and has the advantages of not requiring the specification of the constitutive equation and the narrow gap approximation. The method is applicable to yield stress fluids, but depends on the proper choice of a regularization parameter. Moreover, the extraction of the flow curve and the yield stress value requires a slow iterative procedure [4]. Ancey [4] proposed a method based on a wavelet-vaguelette decomposition for the recovery of the shear flow curve of yield stress materials, such as polymeric gels and granular suspensions. This method does not require knowledge of the constitutive equation and the narrow gap approximation and exhibits greater accuracy and faster convergence than Tikhonov regularization, but requires data filtering. Hoog and Anderssen [15] generalized the Euler-Maclaurin sum formula solution of the Couette viscometry equation introduced by Krieger and Elrod [16] deriving simple and more accurate formulas which are exact for Newtonian fluids and do not involve a numerical differentiation. They also address the issue of flow curve recovery from wide gap rheometry measurements.

More recently, Estellé et al. [3, 17] proposed a simpler procedure based on the Bingham approximation and the use of the maximization of the dissipation of energy to discriminate between the partially sheared gap solution and the fully sheared one, which does not require a priori knowledge of the yield stress nor the evaluation of the flow field in the gap region. This procedure was successfully applied for computing the shear flow curves of model and real fluids for both the Herschel-Bulkley and the Casson models from torque-rotational velocity data in a Couette rheometer. Heirman et al. [5] proposed an integration approach of the Couette inverse problem in order to convert torque measurements into shear flow curve data and describe the Herschel-Bulkley behavior of self-compacting concrete observed in a wide-gap Couette rheometer. Kelessidis and Maglione [1,18] also presented another series expansion methodology to invert the flow equation of a Herschel-Bulkley fluid in Couette concentric cylinder geometry, thus enabling simultaneous computation of the true shear rates and of the three Herschel-Bulkley rheological parameters.

The objective of the present work is to analyze from a different perspective the errors introduced by the standard Newtonian and power-law assumptions used in determining the material properties of viscoplastic fluids from circular Couette viscometric data. For that purpose, the Herschel-Bulkley constitutive equation is employed. In an early study, Darby [19] showed that the error involved in using the local power-law approximation for the shear rate in a Couette viscometer for materials with a yield stress depends upon the constitutive equation of the material, as well as the gap width and the stress level in the gap relative to the yield stress. He found in particular that the error for the Casson fluid is less than half that for the Bingham material. Kelessidis and Maglione [1,18] also calculated the errors resulting from the commonly made Newtonian shear rate assumption in the case of narrow gap Couette viscometers. Their results indicate that significant differences exist between the yield stress and the flow behavior index computed using the Herschel-Bulkley shear rate versus the parameters obtained using its Newtonian counterpart.

In Section 2, the governing equations and some general results are presented. In Section 3, we study the case of a partially yielded (i.e. partially sheared) fluid and present numerical as well analytical results for certain values of the power-law exponent. In Section 4, the case of a fully-yielded (i.e. fully sheared) fluid is analyzed. In Section 5, numerical results are presented for the ratio of the (exact) wall shear rate to its approximation under the power-law assumption and the effects of the gap size, the yield stress, and the power-law exponent are studied. In agreement with previous works [18, 19], the Newtonian and the power-law assumptions for the shear rate lead to significant errors if the fluid obeys a viscoplastic constitutive equation, such as the Herschel-Bulkley model. These errors increase with the gap size and the yield stress value. The conclusions are summarized in Section 6.

2 GOVERNING EQUATIONS

We consider the circular Couette flow between two co-axial long cylinders of radii R_1 and R_2 with $R_1 < R_2$. The inner cylinder is rotating at a constant angular velocity Ω and the outer cylinder is fixed, as illustrated in Figure 1. We assume the following: (a) the flow is steady, isothermal, and laminar, (b) the flow is axisymmetric and end effects are negligible or eliminated by geometrical means, and (c) gravity is negligible. Under these assumptions the flow in cylindrical coordinates is one-dimensional and only the ı-component of the velocity is nonzero, i.e. $u_\theta = u_\theta(r)$. The governing θ-momentum equation becomes

$$\tau_{r\theta} = -\frac{c}{r^2} \tag{1}$$

where c is a constant to be determined from the boundary conditions. For a generalized-Newtonian fluid, the shear stress $\tau_{r\theta}$ takes the form

$$\tau_{r\theta} = \eta(\dot{\gamma}) r \frac{d}{dr}\left(\frac{u_\theta}{r}\right) \tag{2}$$

where η is the shear-rate-dependent viscosity and $\dot{\gamma}$ is the shear rate, given by:

$$\dot{\gamma} = r\left|\frac{d}{dr}\left(\frac{u_\theta}{r}\right)\right| = -r\frac{d}{dr}\left(\frac{u_\theta}{r}\right) \tag{3}$$

(u_θ is a decreasing function of r). The Herschel-Bulkley constitutive equation can be written in the scalar form

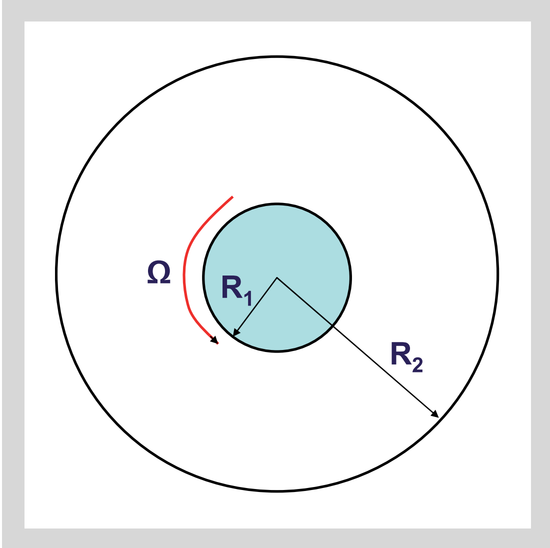


Figure 1: Geometry of circular Couette flow.

$$\begin{cases} \dot{\gamma} = 0, & \tau \leq \tau_o \\ \tau = \tau_o + k\dot{\gamma}^n, & \tau \geq \tau_o \end{cases} \tag{4}$$

where τ is the magnitude of the stress tensor, τ_o is the yield stress, $\dot{\gamma}$ is the magnitude of the rate-of-strain tensor, k is a consistency index and n is the power-law exponent. With $n = 1$ the Bingham fluid is recovered and with $\tau_o = 0$ one gets the power-law model. Thus, for the Herschel-Bulkley fluid, the viscosity is given by:

$$\eta(\dot{\gamma}) = \frac{\tau_o}{\dot{\gamma}} + k\dot{\gamma}^{n-1} \tag{5}$$

Thus, from Eqs. 1, 2, and 5 we get:

$$\dot{\gamma} = \frac{1}{k^{1/n}}\left(\frac{c}{r^2} - \tau_o\right)^{1/n} \tag{6}$$

The governing equations are nondimensionalized scaling r by R_1, u_θ by ΩR_1, $\dot{\gamma}$ by Ω, and τ by τ_o. For the sake of simplicity, the same symbols are used for the dimensionless variables hereafter. The dimensionless forms of Eqs. 4 and 6 are respectively

$$\begin{cases} \dot{\gamma} = 0, & \tau \leq 1 \\ \tau = 1 + \frac{1}{Bn}\dot{\gamma}^n, & \tau \geq 1 \end{cases} \tag{7}$$

and

$$\dot{\gamma} = -r\frac{d}{dr}\left(\frac{u_\theta}{r}\right) = Bn^{1/n}\left(\frac{c}{r^2} - 1\right)^{1/n} \tag{8}$$

where Bn is the generalized Bingham number defined by

$$Bn \equiv \frac{\tau_o}{k\Omega^n} \tag{9}$$

Another assumption we made is that the no-slip condition holds at the two cylinders. Integrating Eq. 8 and applying the boundary condition $u_\theta(1) = 1$ (at the rotating cylinder) lead to the general solution

$$u_\theta(r) = \left[1 - Bn^{1/n}\int_1^r \frac{1}{\xi}\left(\frac{c}{\xi^2} - 1\right)^{1/n} d\xi\right] r \tag{10}$$

The wall shear rate (on the rotating cylinder) is given by

$$\dot{\gamma}_w = Bn^{1/n}\left(c - 1\right)^{1/n} \tag{11}$$

The constant c is determined by applying the remaining boundary condition. With fluids with a yield stress, there are two possible cases for the Couette flow. If the yield stress is exceeded everywhere in the gap, then all the fluid is sheared; otherwise, only part of the fluid is sheared. In the former case, c is calculated by applying the boundary condition $u(R_2) = 0$ at the outer cylinder. Otherwise, the boundary condition used is $u(r_o) = 0$, where r_o ($1 < r_o < R_2$) is the yield point beyond which the fluid is not sheared. The two cases are examined separately in Sections 3 and 4. It can be easily shown that the ratios of the wall shear rate for the Herschel-Bulkley fluid to its power-law and Newtonian counterparts are:

$$\frac{\dot{\gamma}_w}{\dot{\gamma}_{PL}} = \frac{nBn^{1/n}\left(R_2^{2/n} - 1\right)\left(c - 1\right)^{1/n}}{2R_2^{2/n}} \tag{12}$$

and

$$\frac{\dot{\gamma}_w}{\dot{\gamma}_N} = \frac{Bn^{1/n}\left(R_2^2 - 1\right)\left(c - 1\right)^{1/n}}{2R_2^2} \tag{13}$$

3 PARTIALLY-YIELDED FLUID

We consider here the case where the fluid is partially yielded in the region $1 \le r \le r_o < R_2$. At the yield point, r_o, both the shear rate and the velocity vanish. Since $\dot{\gamma} = 0$, for $r = r_o$ we get $c = r_o^2$ from Eq. 8 and the general solution becomes ($1 \le r \le r_o$)

$$u_\theta(r) = \left[1 - Bn^{1/n}\int_1^r \frac{1}{\xi}\left(\frac{r_o^2}{\xi^2} - 1\right)^{1/n} d\xi\right] r \tag{14}$$

where

$$Bn^{1/n} = \frac{1}{\displaystyle\int_1^{r_o} \frac{1}{\xi}\left(\frac{r_o^2}{\xi^2} - 1\right)^{1/n} d\xi} \tag{15}$$

The ratios of the wall shear rate to its power-law and Newtonian counterparts are respectively

$$\frac{\dot{\gamma}_w}{\dot{\gamma}_{PL}} = \frac{nBn^{1/n}\left(R_2^{2/n} - 1\right)\left(r_o^2 - 1\right)^{1/n}}{2R_2^{2/n}} \tag{16}$$

and

$$\frac{\dot{\gamma}_w}{\dot{\gamma}_N} = \frac{Bn^{1/n}\left(R_2^2 - 1\right)\left(r_o^2 - 1\right)^{1/n}}{2R_2^2} \tag{17}$$

In the general case, for a given value of r_o with $1 < r_o < R_2$, one can use numerical integration to find the corresponding Bingham number from Eq. 15 and then calculate the velocity profile from Eq. 14. For certain values of the power-law exponent the integration can be carried out analytically. The results for three such cases, i.e. $n = 1$ (Bingham plastic), 1/2 and 1/3 are listed below.

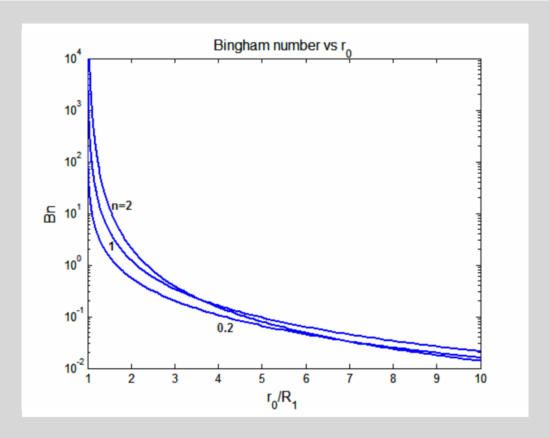
Bingham plastic (n = 1). In this case $Bn \equiv \tau_o/k\Omega$ and

$$u_\theta(r) = r\left[1 - Bn\left\{\frac{r_o^2}{2}\left(1 - \frac{1}{r^2}\right) - \ln r\right\}\right] \tag{18}$$

for $1 \le r \le r_o$ with

$$Bn = \frac{2}{r_o^2 - 2\ln r_o - 1} \tag{19}$$

The ratio of the wall shear rate to its Newtonian counterpart is



$$\frac{\dot{\gamma}_W}{\dot{\gamma}_N}=\frac{Bn\left(R_2^2-1\right)\left(r_o^2-1\right)}{2R_2^2} \tag{20}$$

Herschel-Bulkley fluid with n = 1/2.
Here $Bn \equiv \tau_o/k\Omega^{1/2}$ and

$$u_\theta\left(r\right)=r\left[1-Bn^2\left\{\ln r-r_o^2\left(1-\frac{1}{r^2}\right)+\frac{r_o^4}{4}\left(1-\frac{1}{r^4}\right)\right\}\right] \tag{21}$$

for $1 \le r \le r_o$ with

$$Bn^2=\frac{4}{-4r_o^2+r_o^4+4\ln r_o+3} \tag{22}$$

It turns out that

$$\frac{\dot{\gamma}_W}{\dot{\gamma}_{PL}}=\frac{Bn^2\left(R_2^4-1\right)\left(r_o^2-1\right)^2}{4R_2^4} \tag{23}$$

Herschel-Bulkley fluid with n = 1/3. The Bingham number is defined by $Bn \equiv \tau_o/k\Omega^{1/3}$ and

$$u_\theta\left(r\right)=r\left[1-Bn^3\left\{-\ln r-\frac{3}{2}r_o^2\left(1-\frac{1}{r^2}\right)+\frac{3r_o^4}{4}\left(1-\frac{1}{r^4}\right)+\frac{r_o^6}{6}\left(1-\frac{1}{r^6}\right)\right\}\right] \tag{24}$$

for $1 \le r \le r_o$ with

$$Bn^3=\frac{12}{2r_o^6-9r_o^4+18r_o^2-12\ln r_o-11} \tag{25}$$

In this case, one gets:

$$\frac{\dot{\gamma}_W}{\dot{\gamma}_{PL}}=\frac{Bn^3\left(R_2^6-1\right)\left(r_o^2-1\right)^3}{6R_2^6} \tag{26}$$

In Figure 2, the Bingham number is plotted as a function of the yield point r_o for three different values of the power-law index: $n = 0.2, 1$ and 2. All curves have been calculated numerically as explained above. The results for $n = 1$ (Bingham plastic) can also be obtained directly from Eq. 19. It is clear in Figure 2 as well as from Eq. 15 or from Eqs. 19, 22, and 25, that r_o and, thus, the size of the unsheared region depends on the Bingham number and the power-law exponent. This region increases in size as the Bingham number is increased. As shown in Figure 2, the effect of the power-law exponent changes depending on the value of the Bingham number. We observe that the curve corresponding to the shear thickening fluid ($n = 2$) is above the other two curves ($n = 0.2$ and 1) only for radii ratios less than 6. In Figure 3, the velocity profiles for four values of the power-law index ($n = 1, 1/2, 1/3$, and $3/2$) and various Bingham numbers are shown. In the general case the velocity profiles are calculated by numerical integration using Eq. 14. Here we use numerical integration only for $n = 3/2$. For $n = 1$, 1/2 and 1/3 we have used Eqs. 18, 21, and 24, respectively. The results for the wall shear rate will be discussed below together with the results for the fully-yielded flow.

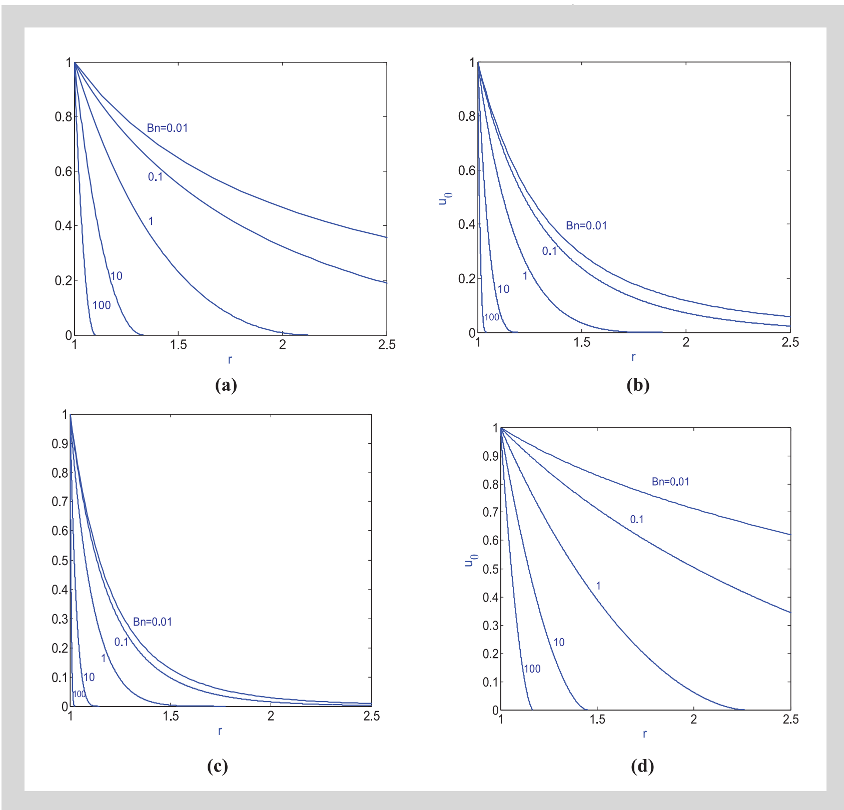


Figure 2 (left): The Bingham number as a function of the dimensionless yield point when the fluid is partially yielded.

Figure 3: Velocity profiles for different Bingham numbers when the fluid is partially yielded: (a) n = 1, (b) n = 1/2, (c) n = 1/3, and (d) n = 3/2.

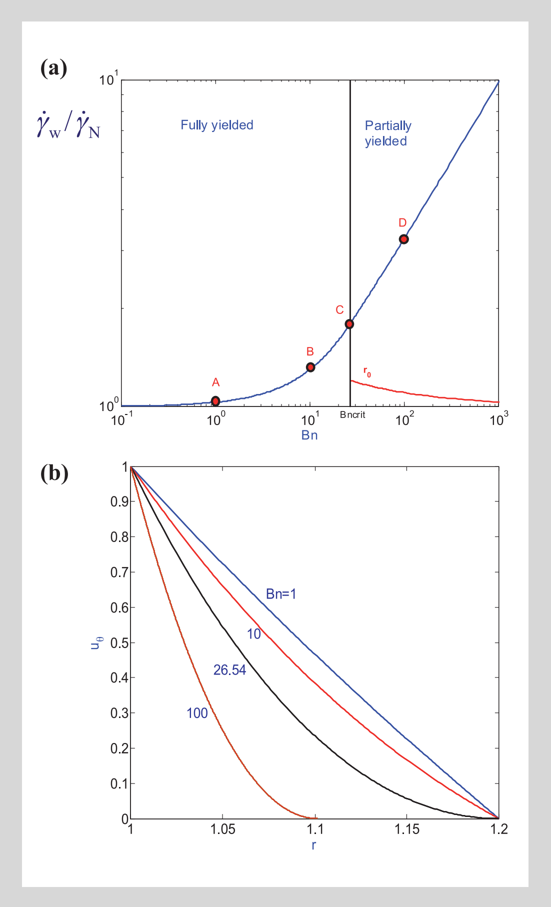
4 FULLY-YIELDED FLUID

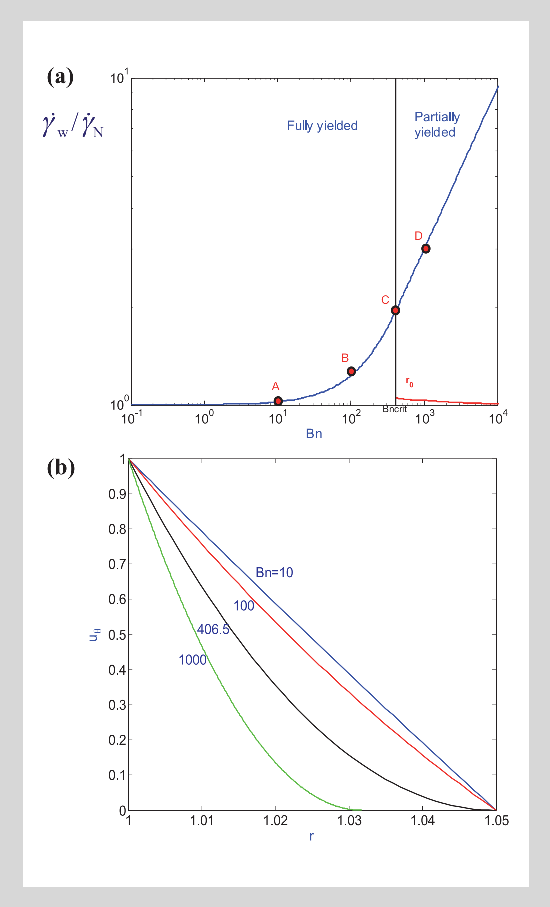
When all the fluid is yielded, the angular velocity is forced to vanish at $r = R_2$. The velocity for $1 \le r \le R_2$ is given by Eq. 10 where now

$$Bn^{1/n}=\frac{1}{\int_1^{R_2}\frac{1}{\xi}\left(\frac{c}{\xi^2}-1\right)^{1/n}d\xi} \tag{27}$$

Figure 4 (left):
Results for Bingham flow ($n = 1$) with $R_2/R_1 = 1.2$: (a) Wall shear rate ratio, $\dot{\gamma}_w/\dot{\gamma}_N$, calculated from Eqs. 31 and 20. The dotted line shows the critical Bingham number above which the fluid is partially yielded. (b) Velocity profiles corresponding to points A, B and C (fully yielded fluid) and D (partially yielded fluid).

Figure 5:
Results for Bingham flow ($n = 1$) with $R_2/R_1 = 1.05$: (a) Wall shear rate ratio, $\dot{\gamma}_w/\dot{\gamma}_N$, calculated from Eqs. 31 and 20. The dotted line shows the critical Bingham number above which the fluid is partially yielded. (b) Velocity profiles corresponding to points A, B and C (fully yielded fluid) and D (partially yielded fluid).





For a given Bingham number, one can find the constant c by solving Eq. 40. The critical Bingham number, Bn^{crit}, below which the fluid is fully yielded corresponds to $c = R_2^2$. Hence, Bn^{crit} is found from

$$\left(Bn^{crit}\right)^{1/n} = \frac{1}{\int_1^{R_2} \frac{1}{\xi}\left(\frac{R_2^2}{\xi^2}-1\right)^{1/n} d\xi} \tag{28}$$

For the cases $n = 1$, 1/2 and 1/3 the critical Bingham number can be calculated from Eqs. 19, 22, and 25 by simply replacing r_o by R_2. The results for $n = 1$ and 1/2 are discussed below. (The problem becomes very cumbersome for $n = 1/3$ and this case is not considered.)

Bingham fluid (n = 1). In this case, one gets the following expression for the velocity

$$u_\theta(r) = \left[1 + Bn\left\{\ln r + \frac{c}{2}\left(\frac{1}{r^2}-1\right)\right\}\right] r \tag{29}$$

where

$$c = \frac{2R_2^2}{R_2^2-1}\left(\ln R_2 + \frac{1}{Bn}\right) \tag{30}$$

and

$$\frac{\dot{\gamma}_w}{\dot{\gamma}_N} = 1 + Bn \ln R_2 - \frac{Bn\left(R_2^2-1\right)}{2R_2^2} \tag{31}$$

Herschel-Bulkley fluid with n = 1/2. In the case of Herschel-Bulkley fluid with $n = 1/2$,

$$u_\theta(r) = \left[1 - Bn^2\left\{\frac{c^2}{4}\left(1-\frac{1}{r^4}\right) - c\left(1-\frac{1}{r^2}\right) + \ln r\right\}\right] r \tag{32}$$

where c is given by

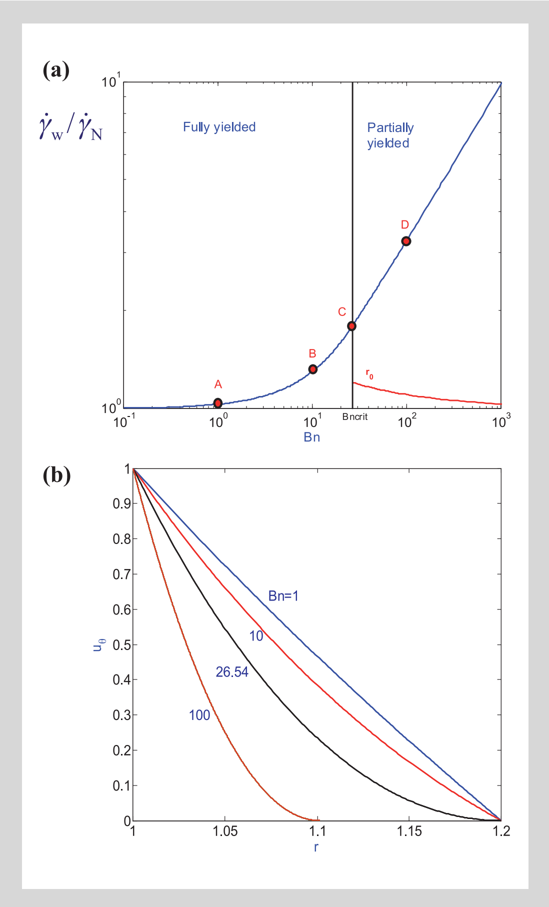
$$c = \frac{2R_2^2}{R_2^2+1}\left[1 + \sqrt{1 + \frac{1+R_2^2}{1-R_2^2}\left(\ln R_2 - \frac{1}{Bn^2}\right)}\right] \tag{33}$$

The ratio of the wall shear rate to its power-law counterpart is:

$$\frac{\dot{\gamma}_w}{\dot{\gamma}_{PL}} = \frac{Bn^2(c-1)^2\left(R_2^4-1\right)}{4R_2^4} \tag{34}$$

5 NUMERICAL RESULTS

In the general case of a fully-yielded fluid, for a given Bingham number the constant c is found by solving Eq. 27 using the bisection method along with numerical integration, (Simpson's



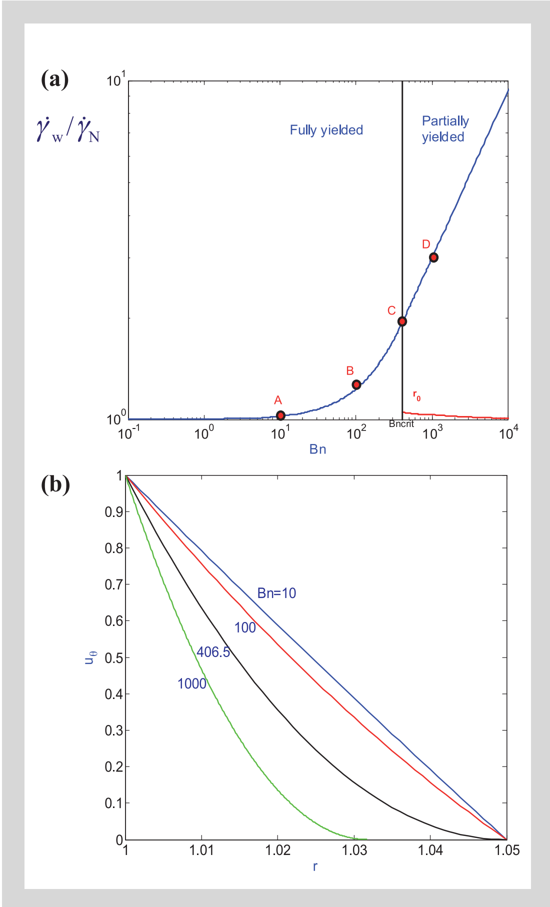


Figure 6 (left): Results for Herschel-Bulkley flow (n = 0.5) with R_2/R_1 = 1.2: (a) Wall shear rate ratio, $\dot{\gamma}_w/\dot{\gamma}_{PL}$, calculated from Eqs. 34 and 23. The dotted line shows the critical Bingham (Bn^{crit} = 9.658) number above which the fluid is partially yielded. (b) Velocity profiles in both the fully- and partially-yielded regimes.

Figure 7: Results for Herschel-Bulkley flow (n = 0.5) with R_2/R_1 = 1.05: (a) Wall shear rate ratio, $\dot{\gamma}_w/\dot{\gamma}_{PL}$, calculated from Eqs. 34 and 23. The dotted line shows the critical Bingham (Bn^{crit} = 77.45) number above which the fluid is partially yielded. (b) Velocity profiles in both the fully- and partially-yielded regimes.

rule with 1001 points). Then, the angular velocity $u_\theta(r)$ is calculated from Eq. 10 using again numerical integration (for a given value of r).

Figures 4 and 5 show results for the wall shear ratio, $\dot{\gamma}_w/\dot{\gamma}_N$, in the case Bingham-plastic flow obtained with R_2/R_1 = 1.2 and 1.05, respectively. In Figure 4a, the wall shear ratio is plotted versus the Bingham number. The dotted line corresponds to the critical Bingham number (Bn^{crit} = 26.54) beyond which the flow is partially yielded. The left part of the curve corresponding to the fully-yielded case ($Bn \leq Bn^{crit}$) was calculated using Eq. 31, while for the right part corresponding to the partially yielded case ($Bn \geq Bn^{crit}$) Eq. 20 was used. The yield point, r_o, calculated from Eq. 19, is also plotted beyond Bncrit. We observe that the wall shear rate ratio increases fast even for small and moderate Bingham numbers below Bn^{crit}. At Bn^{crit} the error is about 18%. In Figure 4b, representative velocity profiles corresponding to points A, B, C, and D of Figure 4a located in both the fully- and partially yielded regimes are shown, which were calculated using Eqs. 29 and 18, respectively. Similar results have been obtained in the case of a smaller radii ratio R_2/R_1 = 1.05 which are shown in Figure 5. However, the increase of the wall shear rate ratio becomes slower as the ratio R_2/R_1 is decreased, in agreement with previous reports [18,19].

Figures 6 and 7 show results obtained for a Herschel-Bulkley fluid with n = 0.5. The calculated wall shear rate ratios, $\dot{\gamma}_w/\dot{\gamma}_{PL}$, for R_2/R_1 = 1.2 and 1.05 respectively are shown in Figures 6a and 7a and representative velocity profiles in both the fully- and partially yielded regimes are given in Figures 6b and 7b. Again the wall shear rate ratio increases with the radii ratio. Of most interest, however, is the fact that the wall shear rate ratio increases as the power-law exponent is reduced.

6 CONCLUSIONS

We have solved the steady-state, one-dimensional circular Couette flow of a Herschel-Bulkley fluid assuming that the inner cylinder is rotating at constant speed while the outer one is fixed. Analytical solutions are presented for certain values of the power-law exponent. In the general case, the momentum equation is integrated numerically. In agreement with previous works [17, 18], the Newtonian and the power-law assumptions for the wall shear rate lead to significant errors which increase with gap size and yield stress. Our results show that the error in the estimated viscosity, which is insignificant when the diameter ratio is close to unity, may grow large depending on the radii ratio, the yield stress, and the power-law exponent. The comparison of the present theoretical results with experimental data and/or data produced following other methodologies is the subject of our current work.

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Corrigendum to “Wall shear rates in circular Couette flow of a Herschel-Bulkley fluid”, 19:3, 34288 (2009)

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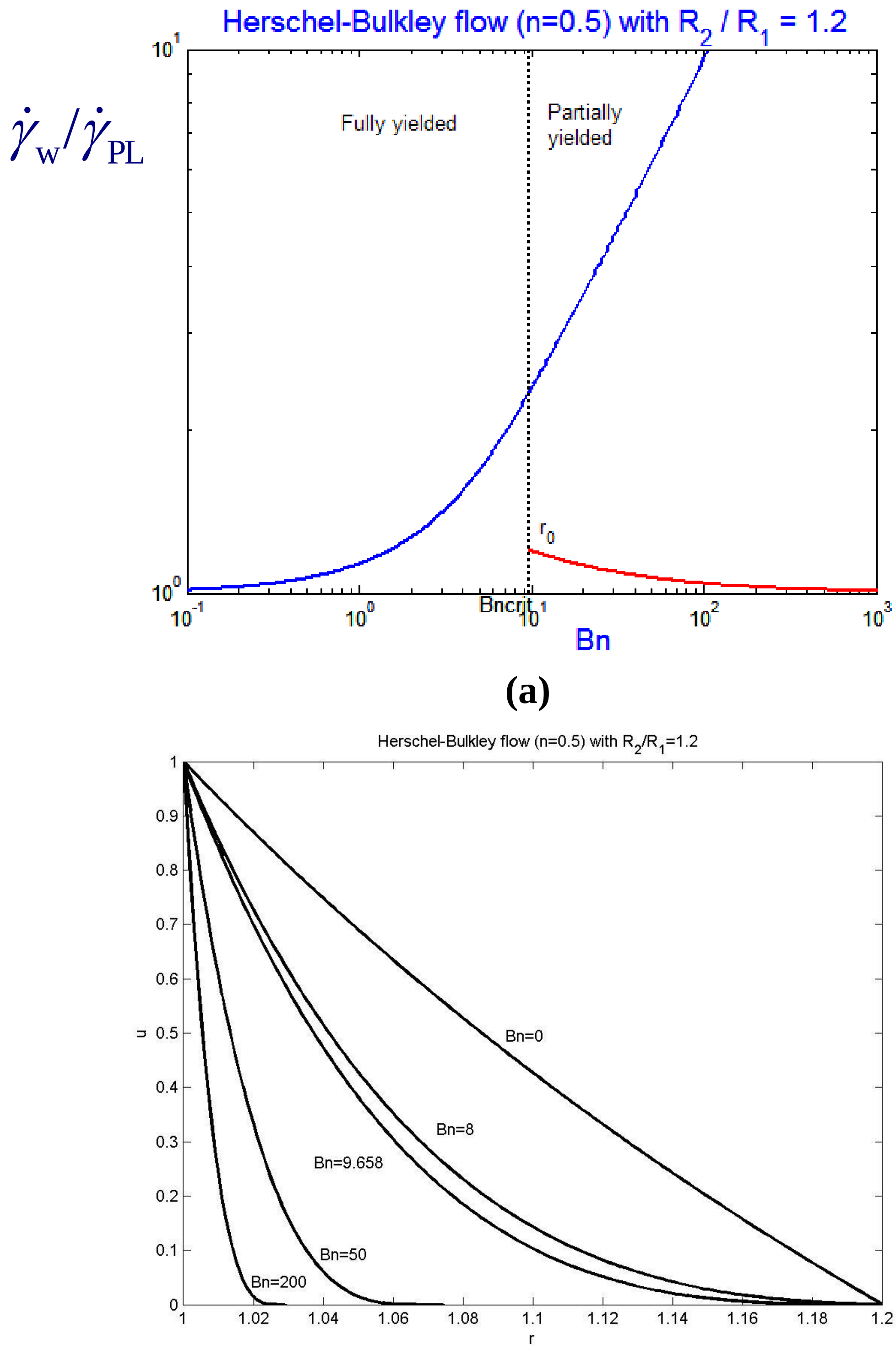
While the captions are correct, Figs. 6 and 7 in the published version of the paper [1] have been accidentally replaced by Figs. 4 and 5, respectively. The correct Figs. 6 and 7 are provided here.

We thank Dr. Gerry Meeten (Schlumberger Cambridge Research) for pointing out the mistake.

Reference

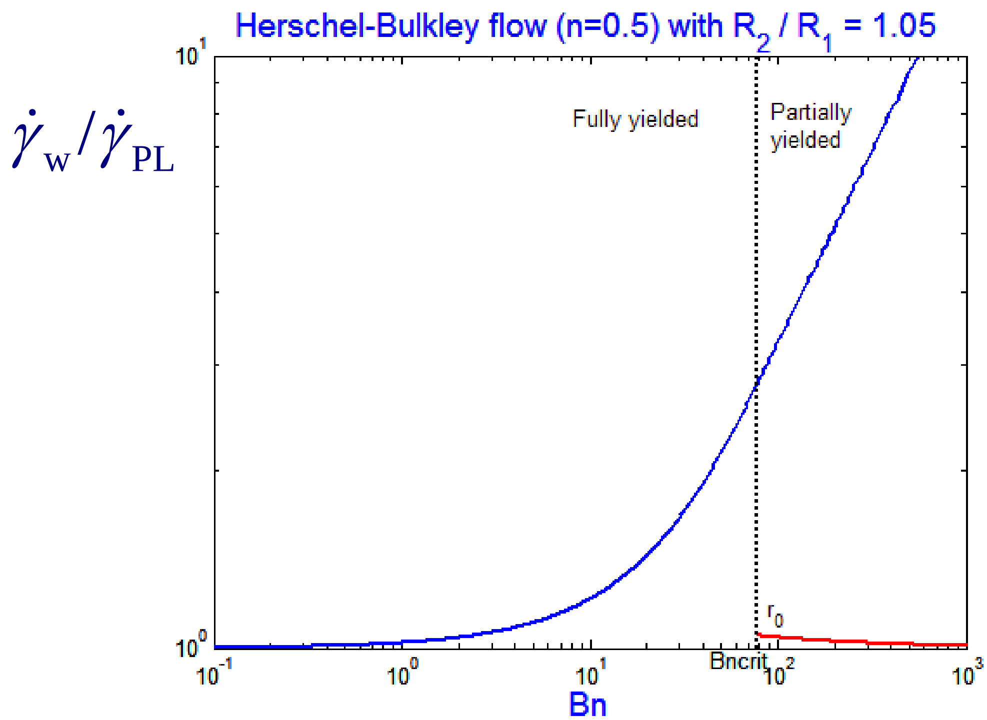
1. Chatzimina M, Georgiou G, Alexandrou AN, Circular Couette flows of viscoplastic fluids, *Appl. Rheol.* **19:3** (2009) 34288.



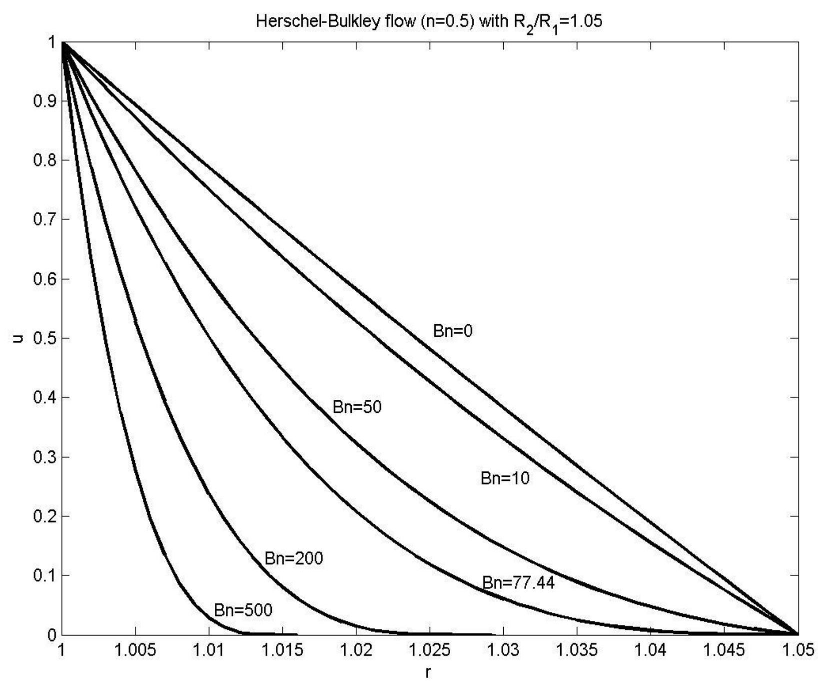
(a)

(b)

Figure 6. Results for Herschel-Bulkley flow (n=0.5) with R_2/R_1=1.2: (a) Wall shear rate ratio, $\dot\gamma_w/\dot\gamma_{PL}$, calculated from Eqs. (53) and (33). The dotted line shows the critical Bingham (Bn^{crit}=9.658) number above which the fluid is partially yielded. (b) Velocity profiles in both the fully- and partially-yielded regimes.



(a)



(b)

Figure 7. Results for Herschel-Bulkley flow (n=0.5) with R_2/R_1=1.05: (a) Wall shear rate ratio, $\dot{\gamma}_w/\dot{\gamma}_{PL}$, calculated from Eqs. (53) and (33). The dotted line shows the critical Bingham (Bn^{crit}=77.45) number above which the fluid is partially yielded. (b) Velocity profiles in both the fully- and partially-yielded regimes.