

## Squeeze Flow of Thixotropic Semisolid Slurries

Andreas N. Alexandrou<sup>1,a\*</sup>, Georgios C. Florides<sup>2,b</sup> and Georgios Georgiou<sup>3,c</sup>

<sup>1</sup>Department of Mechanical and Manufacturing Engineering,  
University Cyprus, P.O. Box 20537, 1678 Nicosia, Cyprus

<sup>2</sup>Department of Mechanical and Manufacturing Engineering,  
University Cyprus, P.O. Box 20537, 1678 Nicosia, Cyprus

<sup>3</sup>Department of Mathematics and Statistics,  
University Cyprus, P.O. Box 20537, 1678 Nicosia, Cyprus

<sup>a</sup>andalex@ucy.ac.cy, <sup>b</sup>gcflorides@eac.com.cy, <sup>c</sup>georgios@ucy.ac.cy

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**Abstract.** An essential element for the integration of a semisolid process in the production of complex commercial components is the availability of accurate mathematical and computational tools that could describe both the rheological behavior and the material characteristics of the suspension, which are strongly affected from its internal structure and its evolution during deformation. In this study we considered the squeeze flow experiment, which is a standard method used to determine material properties of semisolid slurries, where a fixed amount of material is compressed from its topside either under constant load or constant velocity, while the bottom side remains fixed. Through high fidelity computational modeling we simulated the classical compression experiment by including the effects of thixotropy in order to demonstrate its role in determining material constants. More specifically a structural viscoplastic model based on the Bingham plastic constitutive equation is proposed. The yield stress is assumed to vary linearly with the structural parameter which follows a first-order rate equation accounting for the material structure break-down and build-up. The development of the yielded/unyielded regions in relation to material structural changes is analyzed. Furthermore, we performed also simulations, where the compression is interrupted for a short time, in order to study the material internal structure after a short relaxation time.

### Introduction

The motivation for this work derives from our interest in the technology of processing metal slurries in their semisolid state for the production of consistently high integrity parts. These are relatively dense suspensions exhibit yield stress, i.e. they flow only if a finite stress value is exceeded; otherwise, they behave as solids and they show thixotropic behavior with history-dependent material-parameters, i.e. their viscosity decreases with time under constant shearing [1]-[4]. In contrast to other thixotropic materials, semisolid slurries show partially irreversible rheological properties due to the breakage of the welded bonds between the particles [2,5].

The purpose of the present work is to contribute to the use of squeeze flow technique for the rheological characterization of semisolid slurries and the development of a methodology for the determination of their material constants and to investigate numerically the structural changes of a thixotropic yield-stress material in squeeze flow. For the numerical squeeze flow simulation we applied a thixotropic model based on the Bingham plastic constitutive equation with the yield stress depending linearly on a structural parameter [3,6]. It should be noted that while the motivation and most of the literature reviewed above concern the behavior of semisolid slurries, the theory and the model presented below apply to a much larger family of materials.

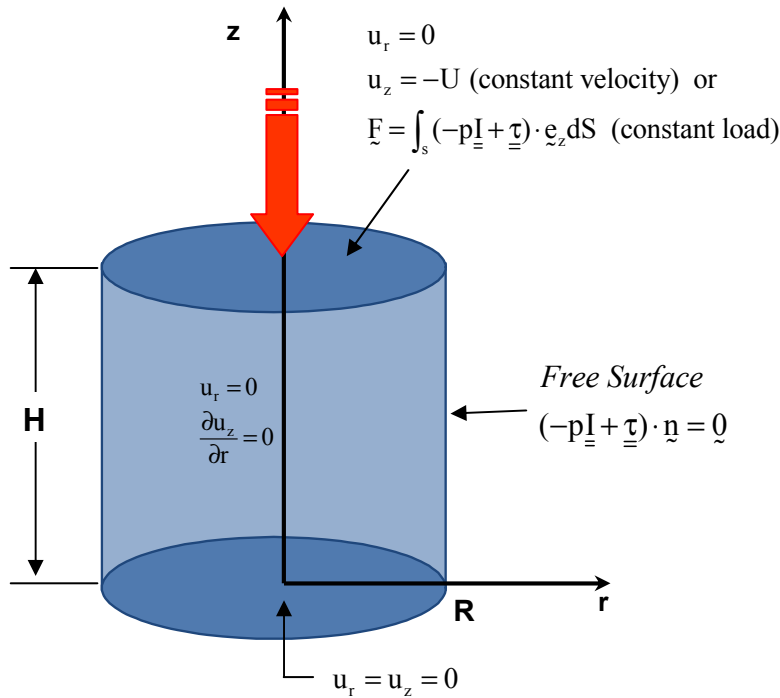
In the following section we present the mathematical formulation for the squeeze flow experiment with the boundary and initial conditions as well as the thixotropic model and the governing equations. Then, after a brief description of the numerical method, we present and discuss preliminary simulations in which the squeeze flow experiment is interrupted for a short period of time.

### Governing equations

The arrangement of the squeeze flow experiment and the boundary conditions of the flow are shown in Figure 1, in which a finite amount of semisolid material is placed between two parallel discs and is then compressed under isothermal conditions from the top with either constant load or constant velocity. The sample is characterized by a cylindrical shape of initial radius  $R_0$  and height  $H_0$ . Symmetric boundary conditions are imposed along the axis of symmetry and the velocity is set to zero along the bottom. On the free surface it is assumed that surface tension is zero. When the sample is compressed at constant load in the direction of gravity, i.e. the dimensionless load is of the form  $\underline{F} = -F \underline{e}_z$ , the dimensionless boundary condition at the top of the sample is given by

$$\int_S (-p\underline{I} + \underline{\tau}) \cdot \underline{e}_z \, dS = -\underline{e}_z, \quad (1)$$

where  $S$  is the surface of the top side of the sample,  $p$  is the pressure,  $\underline{\tau}$  is the viscous stress tensor and  $\underline{I}$  is the unit tensor. As for the initial conditions, the velocity is everywhere set to zero at  $t = 0$ .



**Figure 1:** Geometry and boundary conditions of the compression experiment. At  $t = 0$  the sample is at rest.

In the present work we applied the Bingham constitutive model to describe the viscoplastic behaviour of semisolid slurries, where a finite yield stress has to be exceeded in order for the material to flow:

$$\left. \begin{aligned} \underline{\dot{\gamma}} &= \underline{0}, & \tau < \tau_0 \\ \underline{\tau} &= \left( \frac{\tau_0}{\dot{\gamma}} + \mu \right) \underline{\dot{\gamma}}, & \tau \geq \tau_0 \end{aligned} \right\}, \quad (2)$$

where  $\tau_0$  is the yield stress of the material,  $\dot{\gamma} \equiv \nabla \underline{u} + (\nabla \underline{u})^T$  is the rate of strain tensor,  $u$  is the velocity vector, and the superscript  $T$  denotes the transpose. The magnitudes of  $\dot{\gamma}$  and  $\underline{\tau}$  denoted respectively by  $\dot{\gamma}$  and  $\tau$ , are defined by

$$\tau \equiv \sqrt{\frac{1}{2} \underline{\tau} : \underline{\tau}} = \sqrt{\frac{1}{2} \tau : \tau} \quad \text{and} \quad \dot{\gamma} \equiv \sqrt{\frac{1}{2} \underline{\dot{\gamma}} : \underline{\dot{\gamma}}} = \sqrt{\frac{1}{2} \dot{\gamma} : \dot{\gamma}}, \quad (3)$$

where the symbol  $II$  stands for the second invariant of a tensor.

In the case of squeeze flow under constant velocity  $U$ , we scale the lengths by the initial height  $H_0$ , the velocity  $U$ , the time by  $H_0/U$ , and the pressure  $p$  and the stresses by the yield stress  $\mu U/H_0$ , where  $\mu$  is the plastic viscosity. In the case of squeeze flow under constant load  $F$ , the following velocity scale is used:

$$U = \frac{F}{\mu H_0}. \quad (4)$$

In order to overcome the inherent singularity exhibited by the discontinuous Bingham plastic constitutive model and the associated implementation difficulties in computational codes, we adopt the regularized version of the constitutive equation, as proposed by Papanastasiou [1,8]. The dimensionless form of this equations is

$$\underline{\tau} = \left[ \text{Bn} \frac{1 - \exp(-M\dot{\gamma})}{\dot{\gamma}} + 1 \right] \dot{\gamma}, \quad (5)$$

where

$$\text{Bn} \equiv \frac{\tau_y H_0}{\mu U}, \quad (6)$$

is the Bingham number and

$$M \equiv \frac{mU}{H_0}. \quad (7)$$

is the dimensionless stress growth number.

The flow is governed by the continuity and momentum equations for incompressible flow under zero gravity, the dimensionless forms of which are as follows:

$$\nabla \cdot \underline{u} = 0, \quad (8)$$

$$\text{Re} \frac{D\underline{u}}{Dt} = -\nabla p + \nabla \cdot \underline{\tau}, \quad (9)$$

where

$$\text{Re} \equiv \frac{\rho U H_0}{\mu}. \quad (10)$$

is the Reynolds number and  $D\underline{u}/Dt$  is the material velocity derivative.

**The thixotropic model.** The thixotropic behaviour of the time-dependent, non-linear visco-plastic material is described by a structural parameter model, which has been extensively used in previous works on semisolid slurries and involves a structural parameter,  $\lambda$ , which is a function of time and characterizes the state of the material structure. A fully developed skeleton structure is obtained when  $\lambda=1$  and a completely broken structure when  $\lambda=0$ . The material parameters of the Bingham plastic model, i.e. the plastic viscosity and the yield stress are, in general, functions of  $\lambda$  [2,3,7,9,10]. In the present work, we assume that the plastic viscosity is constant and that the yield stress in Eq. (6) and its regularized version in Eq. (8) vary as follows:

$$\tau_0(t) = \tau_y \lambda(t), \quad (11)$$

where  $\tau_y$  is the yield stress of the fully-structured slurry. Hence the viscous stress tensor is given by

$$\underline{\underline{\tau}} = \left[ Bn \lambda \frac{1 - \exp(-M\dot{\gamma})}{\dot{\gamma}} + 1 \right] \underline{\underline{\dot{\gamma}}}, \quad (12)$$

This simple approach was selected instead of the more complex functional relationship presented by Burgos et al. [10], since the interest here is the very fast structure breakdown associated with the compression test. The evolution of the structural parameter is assumed to follow the first-order rate equation, which in its dimensionless formulation becomes:

$$\frac{D\lambda}{Dt} = a(1-\lambda) - b\lambda\dot{\gamma}e^{c\dot{\gamma}}, \quad (13)$$

where  $a$  is the recovery parameter and  $b$  and  $c$  are the breakdown parameters determined from experimental data. Further information on the non-dimensionalization procedure is presented and discussed by Alexandrou et al. [7]. The two terms in the RHS of Eq. (13) describe the rates of structure build-up and break-down. The exponential in the second term accounts for the fact that the shear stress evolution in shear rate step-up experiments is typically faster than in the step-down one. This is in line with the experimental data of Modigell and Koke [2,11] on semisolid slurries, where a strong dependence of the yield stress on the microstructure and the degree of agglomeration of the solid phase was observed, which is further strengthened with rest time. At steady-state the shear rate is constant and the rates of break-down and build-up are equal. One can then determine the equilibrium value of  $\lambda$ :

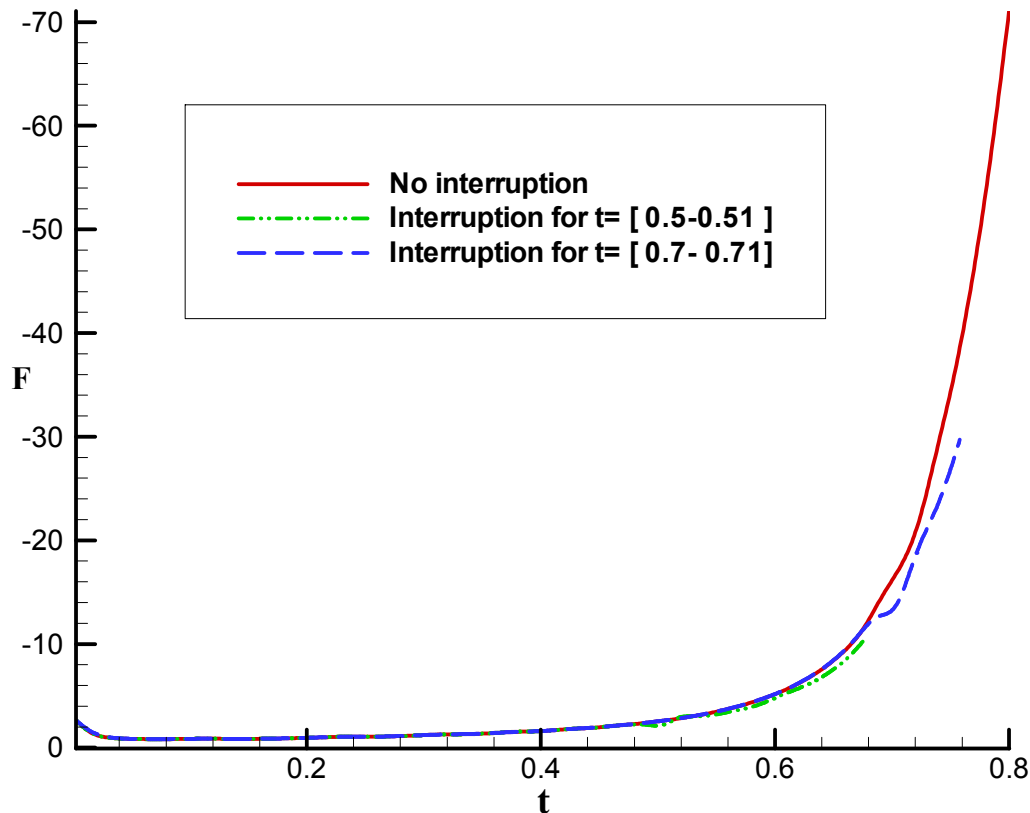
$$\lambda_e = \frac{1}{1 + (b/a)\dot{\gamma}_e e^{c\dot{\gamma}_e}}. \quad (14)$$

### Numerical method

The flow problem is solved in Lagrangian coordinates. The governing equations are discretized using the mixed-Galerkin finite element method with standard nine-node quadrilateral elements for the velocity and four-node ones for the pressure. A Newton-Raphson iteration technique is applied to solve the non-linear system of equations with an error tolerance equal to  $10^{-5}$ . Remeshing is achieved by using a Laplace-type discretization algorithm with an emphasis in the construction for finer mesh at critical corners. Further details on the numerical method can be found in Alexandrou et al. [7].

### Results and Discussion

In this section we present preliminary numerical simulations under constant velocity of a selected base flow ( $Re=1$ ,  $Bn=1$ ,  $a=1$ ,  $b=1$ , and  $c=0.01$ ) [7] in which the sample is squeezed down to a given height and then is left at rest for a specified time, after which compression continues again. More specifically, we present results for two different interruption periods, i.e. for  $t \in [0.50, 0.51]$  and  $t \in [0.70, 0.71]$ . In Fig. 2 we illustrate the effect on the resulting load of these two interrupted cases. The compression appears to stop earlier when an interruption criterion is applied with the necessary load for the compression being reduced. This work is at an early stage and we will carry out more simulation regarding interruptions criteria in our future work. Such experiments have been carried out by Rodts et al. [14] on bentonite suspensions; they reported that starting from different times of rest the material undergoes similar changes in structure and only the force is reduced.

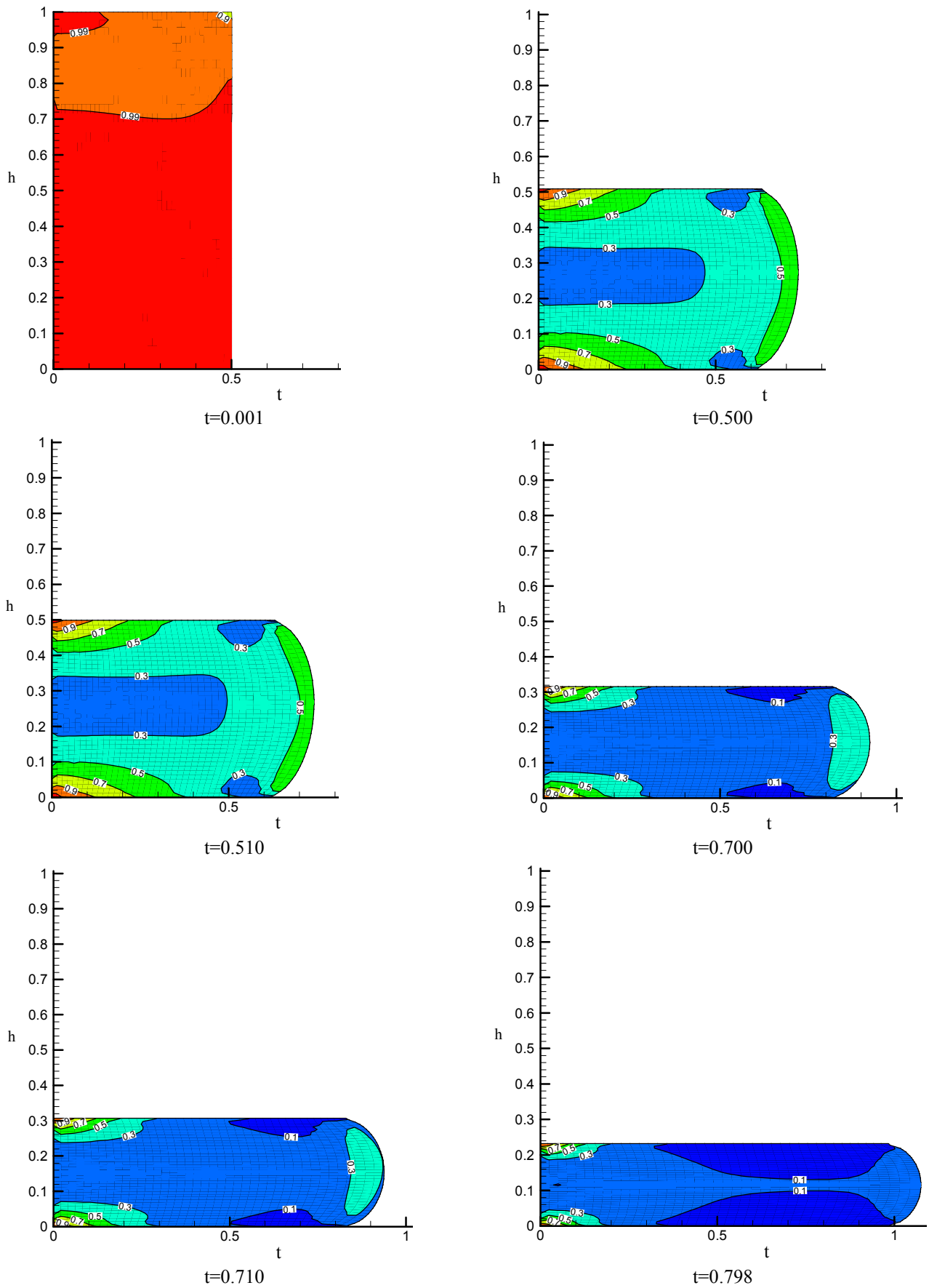


**Figure 2:** Evolution of the resulting load in the three squeeze flow experiments;  $Re=1, Bn=1, a=1, b=1,$  and  $c=0.01$

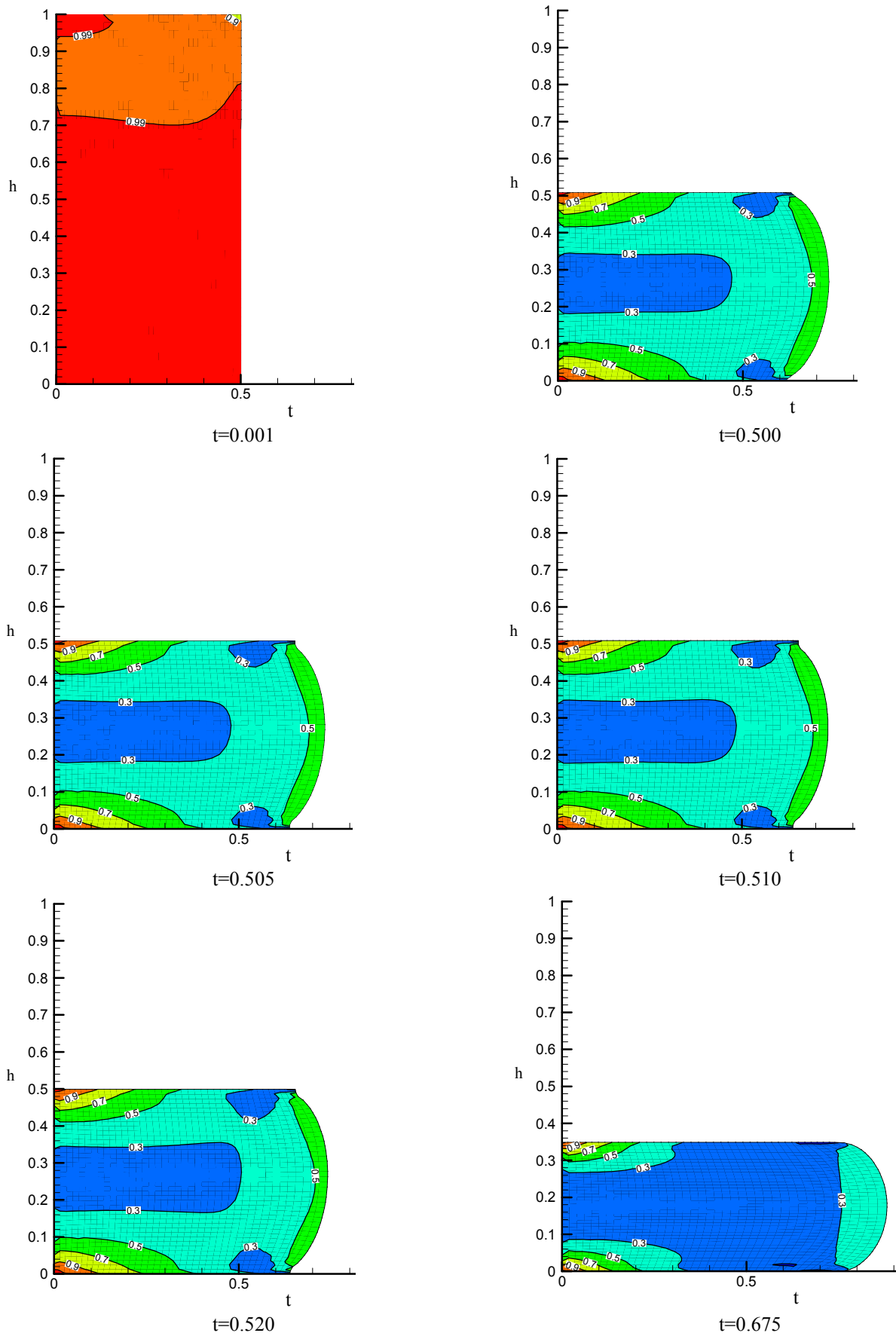
In Figs. 3-5 we show representative snapshots of the evolution of the structural parameter  $\lambda$  during squeeze flow under constant velocity of our the base flow ( $Re=1, Bn=1, a=1, b=1,$  and  $c=0.01$ ) [7] for the three cases of interest: uninterrupted (Fig. 3), interrupted for  $t \in [0.50, 0.51]$  (Fig. 4) and interrupted for  $t \in [0.70, 0.710]$  (Fig. 5). The break-down of the inter-particle bonds begins with the squeeze flow initialization, starting from the upper edge of the sample and spreading towards the axis of symmetry. The average broken bonds and particle rearrangement increase along the whole material with the value of  $\lambda < 0.1$  occurring at the top and the bottom edges near the outer surface. As shown in Figs. 4 and 5 the average broken structure in the sample is reduced when interruption is applied, causing the compression to stop at earlier stage. Although it cannot be clearly observed from the snapshots of Figs. 3-5, an increase in the structural parameter  $\lambda$  during the interruption of the compression is observed, i.e. a structure built-up occurs. As soon as the load is applied again, the structure breakdown continues as the sample is further compressed.

### Concluding remarks

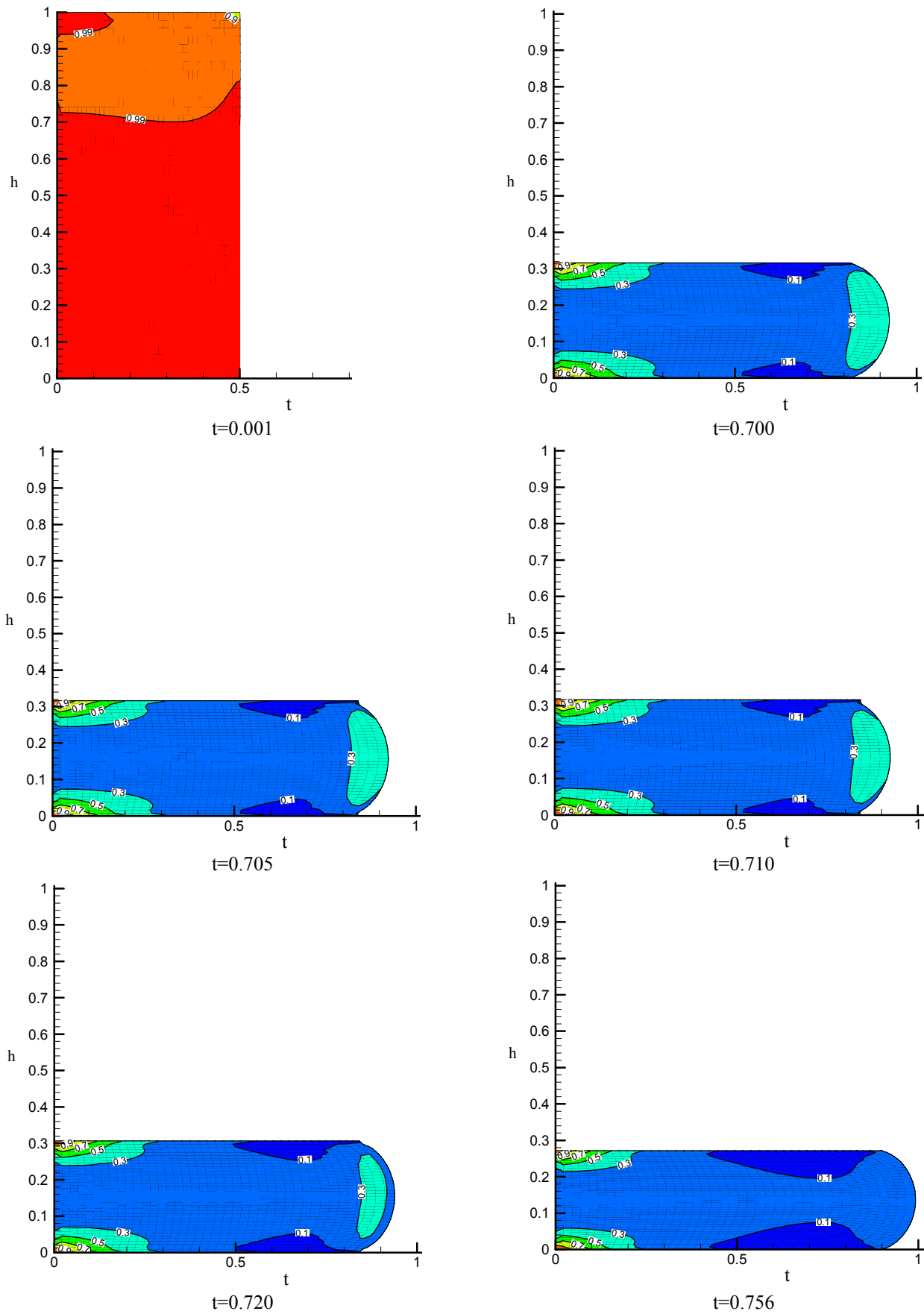
The present numerical simulations are encouraging as they could contribute to the development of a methodology for the determination of the material constants of semisolid slurries used in the processing of alloys, which behave as thixotropic, non-linear visco-plastic material with history-dependent material-parameters. Evidently, further investigation on the material behaviour and its internal structural evolution, when a relaxation time is applied, is needed to compare them with experimental results.



**Figure 3:** Contours of the structural parameter  $\lambda$  during squeeze flow under constant velocity without interruption;  $Re=1, Bn=1, a=1, b=1,$  and  $c=0.01$



**Figure 4:** Contours of the structural parameter  $\lambda$  during squeeze flow under constant velocity with interruption for  $t \in [0.5, 0.51]$ ;  $Re=1$ ,  $Bn=1$ ,  $a=1$ ,  $b=1$ , and  $c=0.01$



**Figure 5:** Contours of the structural parameter  $\lambda$  during squeeze flow under constant velocity with interruption for  $t \in [0.7, 0.71]$ ;  $Re=1$ ,  $Bn=1$ ,  $a=1$ ,  $b=1$ , and  $c=0.01$



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