

Recent progress on invariant Einstein metrics on homogeneous spaces and compact Lie groups

Andreas Arvanitoyeorgos

University of Patras, Department of Mathematics, GR-26500 Rion, Greece

A Riemannian manifold (M, g) is called Einstein if the Ricci tensor satisfies $\text{Ric}(g) = \lambda g$ for some $\lambda \in \mathbb{R}$. General existence results are difficult to obtain, therefore the problem becomes more tractable under symmetry assumptions. Such cases are homogeneous spaces G/H equipped with a G -invariant metric and compact Lie groups G with a left-invariant metric.

The aim of the present talk is to give an overview of recent results on invariant Einstein metrics for compact Lie groups and for an important class of homogeneous spaces the generalized flag manifolds. These are orbits of the adjoint representation of a compact Lie group and diffeomorphic to $G/C(T)$, where T is a torus in G . I will also discuss the problem of finding all invariant Einstein metrics for homogeneous spaces whose isotropy representation contains equivalent summands. This causes a difficulty to describe all invariant metrics. Examples of such spaces are the real Stiefel manifolds $G/K = SO(n)/SO(n-k)$ of orthonormal k -frames in \mathbb{R}^n .

In all the above cases the Einstein equation reduces to an algebraic system of equations which in general is difficult to be so solved. However, we are able to study such systems by using Gröbner bases methods.