The radius of convexity of three kind of normalized Bessel functions of the first kind

Árpád Baricz

Babes-Bolyai University

Cluj-Napoca, Romania

Abstract

In this talk our aim is to determine the radius of convexity for three kind of normalized Bessel functions of the first kind. In the mentioned cases the normalized Bessel functions are starlike-univalent and convex-univalent, respectively, on the determined disks. The key tools in the proofs of the main results are some new Mittag-Leffler expansions for quotients of Bessel functions of the first kind, special properties of the zeros of Bessel functions of the first kind and their derivative, and the fact that the smallest positive zeros of some Dini functions are less than the first positive zero of the Bessel function of the first kind. Moreover, we find the optimal parameters for which these normalized Bessel functions are convex in the open unit disk. In addition, we disprove a conjecture of Baricz and Ponnusamy concerning the convexity of the Bessel function of the first kind.