

# **Ergodic theory and regularity properties of diophantine equations**

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## **Abstract**

An algebraic equation is called partition regular if on any finite partition of the integers the equation can be solved on some cell of the partition. Since the theorems of Schur and van der Waerden dating in the early 1900's, numerous partition regularity results have been proved for linear equations using diverse tools coming from areas such as combinatorics, harmonic analysis, and ergodic theory. Progress has been very scarce for non-linear equations, the hardest case being equations in three variables. In this talk I will go over the history of such problems and will briefly discuss some very recent joint work with Bernard Host where we give the first partition regularity result for some class of homogeneous quadratic equations. The proof has a soft touch of ergodic theory and the backbone of the argument is a decomposition result for multiplicative functions that is of independent number theoretic interest. I will also mention several easy to state open problems.