Continuous models for traffic congestion : equilibrium and optimization, regularity and numerics

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Abstact

I will present in this seminar a quite natural model for equilibrium in traffic congestion situations. The basic idea is that the "cost" (in terms of travel time) for each path depends on how many agents pass along the same path (or part of it) and that if a configuration is an equilibrium, than every pair of points is only connected through paths that are geodesic with respect to this distance given by travel time, a distance depending itself on the distribution of agents on the paths. It is easy to see that this is simply a Nash equilibrium in a game whose players are the agents who need to commute and the payoff is the travel time. We can also see that the equilibrium is characterized by the property of minimizing a functional of the overall cost, which makes it a "potential game". This model, introduced by Wardrop in the 50's in the discrete case (either a finite number or a continuous quantity of players, but on a given finite graph), has been studied in a continuous framework - mainly in collaboration with G. Carlier (Paris-Dauphine) - in recent years, finding some interesting points beyond modelling and the variational aspects. In particular, I will show how this problem is related to a minimal flow problem under divergence constraints, for which many regularity questions are particularly relevant and allow to rigorously analyze the equivalence with the original problem. Finally, I will present a numerical method for finding the optimal configuration (and therefore the equilibrium), based on duality and on the "Fast Marching Method" for computing distances with respect to a non-uniform metric.