One-dimensional symmetry results for entire solutions to semilinear elliptic equations in phase transitions

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Abstract

The problem of showing one-dimensional symmetry of entire solutions to elliptic equations of Allen-Cahn type, modelling phase transitions, has received considerable attention in the last years. The motivation stems mainly from the famous De Giorgi's conjecture (in analogy to the Bernstein problem for minimal graphs) as well as that of Gibbons. Both conjectures have essentially been settled by now. In the first part of the talk, we will review these results.

Gibbons' conjecture implies that if a level set of a solution is bounded from above and below by two parallel hyperplanes then that solution is one-dimensional (and that level set is a hyperplane). In the second part of the talk, we will show that the latter assertion still holds if a level set is bounded only from above (or below) by a hyperplane provided that it touches it at some point. We will also suggest a connection of our result to the strong maximum principle from the theory of minimal surfaces.

If time permits, we will sketch how our approach can be used to improve some results on one-dimensional symmetry of solutions in cylindrical domains provided that the cylinder is convex, and rigidity results for solutions with the same uniform limits at respective infinities.